Workshop on Analysis and Applications

PROGRAM

May 29, 2021

9.00 - 9.10: **Opening**

Chairperson: Anna Rita Sambucini

9.10 - 9.50: **Invited talk**

Jürgen Appell and Simon Reinwand (University of Würzburg, Germany)
Compactness Properties of Two Operators

In our talk we analyze compactness properties of the multiplication operator $M_\mu x(t) := \mu(t)x(t)$ generated by some function $\mu : [0,1] \longrightarrow \mathbb{R}$, and the substitution operator $S_\varphi x(t) := x(\varphi(t))$ generated by some function $\varphi : [0,1] \longrightarrow [0,1]$, in the spaces $C[0,1]$ and $BV[0,1]$. Typical questions we are interested in are:

- Under what condition on μ is the operator M_μ compact?
- Under what condition on φ is the operator S_φ compact?
- Can we express the essential norm $|||M_\mu|||$ in terms of μ?
- Can we express the essential norm $|||S_\varphi|||$ in terms of φ?

A particular emphasis will be put on examples which illustrate our abstract results. This is joint work with Laura Angeloni and Gianluca Vinti (Perugia, Italy), and Tomás Dominguez Benavides (Sevilla, Spain).

9.55 - 10.15: **Gennaro Infante** (Università della Calabria) - *Eigenvalues of Elliptic Functional Differential Systems via a Birkhoff–Kellogg Type Theorem*

Motivated by recent interest on Kirchhoff-type equations, we utilize a classical, yet very powerful, tool of nonlinear functional analysis in order to investigate the existence of positive eigenvalues of systems of elliptic functional differential equations

subject to functional boundary conditions. We obtain a localization of the corresponding non-negative eigenfunctions in terms of their norm. Under additional growth conditions, we also prove the existence of an unbounded set of eigenfunctions for these systems. The class of equations that we study is fairly general and our approach covers some systems of nonlocal elliptic differential equations subject to nonlocal boundary conditions. An example is presented to illustrate the theory.

10.20 - 10.40: **Michele Piconi** (Università degli Studi di Perugia) - *On the convergence properties of Durrmeyer-Sampling Type Operators in Orlicz Spaces*

Durrmeyer-Sampling type operators (DSO) represent a further generalization of the well-known Generalized and Kantorovich-Sampling operators, both introduced as a approximate version of the classical sampling theorem. A modular convergence theorem in Orlicz spaces has been proved in one-dimensional case, by which the convergence in Lp-spaces can be deduced as particular case. This means that the DSO allow to approximate not-necessarily continuous signals: this is crucial mainly from the application point of view, especially in image processing. In the continuous case, a pointwise and uniform convergence theorem has been established, including quantitative estimate for the order of approximation. Moreover, all the above convergence results for DSO can also be extended in the multidimensional setting. By the generality of both the assumptions on moments as the results, several examples for a large class of kernels will be discussed.

10.45 - 11.05: **Luigi Muglia** (Università della Calabria) - *Some results on the approximation of solutions of variational inequalities for multivalued maps on Banach spaces*

Multivalued $*$-nonexpansive mappings are studied in Banach spaces. The demiclosedness principle is established. Here we focus on the problem of solving a variational inequality which is defined on the set of fixed points of a multivalued $*$-nonexpansive mapping. For this purpose, we introduce two algorithms approximating the unique solution of the variational inequality. (Joint work with Giuseppe Marino)

11.10 - 11.20: Break

Chairperson: Luisa Di Piazza

11.20 - 11.40: **Laura Poggiolini** (Università degli Studi di Firenze)
Hamiltonian methods in Optimal Control

We show how Hamiltonian methods allow to provide sufficient second order conditions for strong local optimality of Pontryagin extremals in optimal control problems with a control affine dynamics and bounded controls. If the cost is smooth, then the typical structure of extremal trajectories is the concatenation of bang and singular arcs. In Bolza problems where the integral cost has a L^1-growth with respect to the control, extremal trajectories present also a new kind of arcs, which in the literature are known as "zero arcs" or "inactivated arcs".

11.45 - 12.05: **Zoltan Satmari** (University of Oradea, Romania)
Iterative Bernstein splines technique applied to fractional order differential equations

An iterative numerical method is developed for solving two-point boundary value problems associated to differential equations of fractional order $\alpha \in (0, 1)$. The method is based on a sequence of piecewise Bernstein polynomial functions involved at each iterative step of the Picard iteration applied to the corresponding Volterra-Fredholm integral operator. The convergence of the method is obtained by providing the error estimates in terms of the modulus of continuity. The particular case of initial value problems is discussed. The convergence theoretical result is tested on a numerical experiment revealing the accuracy of the iterative method and a comparison with the performances of the trapezoidal product integration method is presented. (Joint work with Alexandru Mihai Bica)

12.10 - 12.30: **Valeria Marraffa** (Università degli Studi di Palermo)
Differential inclusions: existence results and closure properties

The role of convergence results for integrals in the theory of differential and integral equations is well-known. Indeed, studying a large number of problems one can notice the appearance of discontinuities in the behavior of the functions, so we are lead to the idea of working with measure driven problems, i.e.

$$x(t) = x_0 + \int_0^t f(s, x(s))\, dg(s)$$

where X is a Banach space, g is a real bounded variation function, $x_0 \in X$ and $f : [0, 1] \times X \to X$. It is of interest to develop an existence theory for this kind of

problems when the function g is regulated. It is not an easy task since the properties of primitives with respect to such functions are very weak. It is also important to have closure results for this problem, namely to check if when considering a sequence $(g_n)_n$ of functions converging to a function g the solutions of the equation governed by g_n is close to solutions of the equation governed by g. To this purpose, it is necessary to have a convergence result for Stieltjes integrals and since when working with regulated functions the most appropriate integration theory is the Kurzweil-Stieltjes one, we consider a convergence theorem for the Kurzweil-Stieltjes integral. This convergence result is used to get the existence of regulated solutions for integral equations and inclusions driven by regulated functions in general Banach spaces. Moreover, we focus on the closure properties of the solutions set for such problems; namely, to study if, when taking a sequence of regulated functions $(g_n)_n$ converging to a regulated function g, the solution set of the problem governed by g_n is close to the solution set of the problem governed by g. We consider also non-convex measure differential inclusions

$$\begin{aligned} &dx(t) \in G(t, x(t))\, d\mu_g(t), \\ &x(0) = x_0 \end{aligned}$$

with $x_0 \in \mathbb{R}^d$, under excess bounded variation assumptions on the velocity set $G(t, x(t))$ and make use of interesting selection principles. The map $G : [0, 1] \times \mathbb{R}^d \to \mathcal{P}_k(\mathbb{R}^d)$ has compact possibly non-convex values and $g : [0, 1] \to \mathbb{R}$ is a left-continuous nondecreasing function whose distributional derivative (i.e. the Stieltjes measure generated by g) is denoted by μ_g. Let us remark that it is unnatural to expect the solutions to be absolutely continuous or even continuous, and so, the considered space in which the theory is developed is the space of functions of bounded variation. (Joint work with Luisa Di Piazza and Bianca Satco)

12.35 - 12.55: **Marco Seracini** (Università degli Studi di Perugia)
Sampling type Kantorovich operators for eye fundus image reconstruction

Sampling Kantorovich operators (SKO) have the advantage to extend the results of the classic and generalized sampling theorems to class of not necessarily continuous functions. Their formalization in one dimension has been extended to the multidimensional case, allowing their application to digital images.
Thanks to their low-pass characteristic, SKO can filter undesired noise in images, reducing artifacts, as proved in the solution of both medical as engineering specific applicative problems. Taking into account these results, SKO and other Digital Image Processing techniques have been applied to Optical Coherence Tomography (OCT) data. The results have been the improvement of the visual quality of the images, potentially increasing the diagnostic possibilities.

Chairperson: Carlo Bardaro

15.00 - 15.20: **Marco Spadini** (Università degli Studi di Firenze)
Periodic perturbations of a class of Functional Differential Equations with applications to a HIV model

We study the harmonic response to periodic perturbations of a particular class of functional differential equations that have some physical and biological relevance. Some relations with a well-known model for HIV replication are presented. Our investigation is based on a combination of degree-theoretic methods and a technique that allows to associate the bounded solutions of a functional equation belonging to the class under scrutiny to bounded solutions of a suitable ordinary differential equation.

15.25 - 15.45: **Vittorio Colao** (Università della Calabria)
On the convergence rate of viscosity iterations

Iterative methods for approximating fixed points had been one of the most flourishing topic in recent years. Nevertheless, there is still room for new problems. In this talk, we will focus on a question regarding the convergence rate of certain strongly convergent sequences. We will attempt a partial solution to the problem and we will analyse further extensions.

15.50 - 16.10: **Ilaria Mantellini** (Università degli studi di Perugia)
A class of integral operators that fix exponential functions

We introduce a general class of integral operators that fix exponential functions, containing several recent modified operators of Gauss-Weierstrass, Picard or moment operators. Pointwise convergence theorems are studied, using a Korovkin-type theorem and a Voronovskaja-type formula is obtained.

16.15 - 16.35: **Luca Zampogni** (Università degli Studi di Perugia)
A general method to study the convergence of nonlinear operators in Orlicz spaces

We introduce a general setting in which we define nets of nonlinear operators whose domains are some set of functions defined in a locally compact topological group. We analyze the behavior of such nets, and detect the fairest assumption which are needed for the nets to converge in Orlicz spaces. As a consequence, we give a result of convergence in a subspace of a Orlicz space. (Joint work with Gianluca Vinti)

16.40 - 16.50: Break

Chairperson: Paola Rubbioni

16.50 - 17.10: **Danilo Costarelli** (Università degli Studi di Perugia)
Neural Network operators: approximation results

The theory of neural network operators has been widely studied in last years in view of its connections with the well-known artificial neural networks. In the present talk we will present some recent constructive approximation results for the above family of operators. In particular we will show suitable asymptotic formulas, which are based on the so-called algebraic truncated moments of the density functions (kernels) generated by sigmoidal functions. As a direct consequence of the above asymptotic expansions we can prove some Voronovskaja formulas. Further, also operators with high order convergence have been studied by considering finite linear combination of the above neural network type operators. Several concrete examples of sigmoidal functions will be discussed in details, such as the logistic functions and many others.

17.15 - 17.35: **Laura Angeloni** (Università degli Studi di Perugia)
Estimates in variation for sampling-type operators in multidimensional frame

Working with families of operators in BV-spaces, a classical and important result that is usually investigated is an estimate of the variation of the operators in terms of the variation of the function to which they are applied. We will present estimates of this kind, known in literature as "variation diminishing type estimates", in the multidimensional frame by means of the concept of Tonelli variation, for sampling-type operators. The sampling-type operators represent a widely investigated topic, in the last years, in view of their deep relations with Approximation Theory, Signal and Image Processing, from both theoretical and applicative aspects. In particular, variation-diminishing type results have applications to some problem of Digital Image Processing: such properties, indeed, may be regarded as smoothing procedures. (Joint work with D. Costarelli and G. Vinti)

17.40 - 18.00: **Anna Rita Sambucini** (Università degli Studi di Perugia)
Bartle-Dunford-Schwartz multivalued integral and its Radon-Nikodým representation

An integral for a scalar function with respect to a multimeasure N taking its values in a locally convex space is introduced. The definition is independent of the selections of N and is related to a functional version of the Bartle-Dunford-Schwartz integral with respect to a vector measure presented by Lewis. Its properties are studied together with its application to Radon-Nikodým theorems in order to represent as an integrable derivative the ratio of a general multimeasures; equivalent conditions are provided. (Joint work with L. Di Piazza and K. Musiał)

18.05: **Closure**